OCR Additional Maths Exam Questions - Linear Programming

10 **(i)** Illustrate on one graph the following three inequalities.

$$y \geqslant x - 1$$
$$x \geqslant 2$$
$$2x + y \geqslant 8$$

Draw suitable boundaries and shade areas that are **excluded**. **[4]**

(ii) Write down the minimum value of y in this region. **[1]**

14 A firm has to transport 1500 packages to a site. It has a number of large vans which will transport 200 packages each and a number of small vans which will transport 100 packages each.

Let x be the number of large vans and let y be the number of small vans used.

(i) Write down an inequality based on the number of packages transported. **[2]**

The firm needs to use at least as many small vans as large vans.

(ii) Write a second inequality. **[1]**

(iii) Plot these two inequalities on a graph, using 1 cm to represent one van on each axis. Indicate the region for which these inequalities hold. Shade the area that is **not** required. **[3]**

A large van costs £80 to complete the trip and a small van costs £60 to complete the trip.

(iv) Write down the objective function and hence find from your graph the number of each type of van that will minimise the cost, and work out that cost. **[4]**

(v) What choice of vans should be made to minimise the cost if the restriction about the large and small vans is removed? Work out the cost in this case. **[2]**

8 **(i)** On the axes given, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$5x + 3y \geqslant 30$$
$$3x + y \geqslant 12$$
$$y \geqslant 0$$
$$x \geqslant 0$$

[5]

(ii) Find the minimum value of $6x + y$ subject to these conditions. **[2]**

13 A number of students from a group of 20 boys and 30 girls are to be selected to attend a one-day conference.

The number of girls attending must be at least the same as the number of boys but no more than twice the number of boys.

(i) Let there be x boys and y girls selected.
Given that $x > 0$ and $y > 0$, write down four more inequalities to represent the information. **[3]**

(ii) Plot these inequalities on the grid provided. Indicate the region for which the inequalities hold. Shade the area that is **not** required. **[5]**

(iii) In order to attend the conference the students need to be given a special uniform.
The uniform for the boys costs £40 and the uniform for the girls cost £50. The school has £2000 to spend on the uniforms.

By plotting the appropriate line on your graph, find the maximum number of students that could go to the conference. **[4]**

13 A company needs to buy some storage units. There are two types of unit available, type X and type Y. The cost of each type of unit, the floor space required and the volume for storage are given in the following table.

	Cost per unit (£)	**Floor space required (m^2)**	**Volume for storage (m^3)**
X	100	2	3.5
Y	120	1.5	3

The maximum cost allowed for the purchase of the units is £1200 and the maximum floor space available is $18\,m^2$.
The company wants to maximise the volume for storage.

Let x and y be the number of each type of unit, X and Y, respectively.

(i) Write down an inequality for the total cost and an inequality for the total floor space required. **[3]**

(ii) Draw the inequalities you gave in **(i)** on the grid provided in the answer book. Given that $x \geqslant 0$ and $y \geqslant 0$, indicate the region for which the inequalities hold by shading the area that is **not** required. **[4]**

(iii) Write down the objective function for the volume for storage and find the combination of units that should be bought to maximise the volume for storage.
Write down this maximum volume. **[5]**

14 A company produces bottles of two liquids, X and Y. There are two ingredients, A and B, in each liquid.

The table shows the quantities, in centilitres (cl), of A and B needed for each bottle of liquid.

	A	B
X	4	2
Y	3	5

Each day the company can use 84 cl of A and 90 cl of B.

From this information an analyst writes down the inequality $4x + 3y \leqslant 84$.

(i) Explain what x and y stand for in this inequality and explain what the inequality models. **[2]**

(ii) Use the information given to write down another inequality, other than $x \geqslant 0$ and $y \geqslant 0$. **[1]**

(iii) On the grid given in the answer booklet, illustrate your two inequalities. Shade the region that is not required. **[3]**

(iv) The company needs to produce the same number of bottles of X and of Y each day.

Find the maximum number of bottles of X and of Y that the company can produce. **[2]**

(v) On one day the company does not have to produce the same numbers of bottles of X and of Y.

Write down the maximum number of bottles that can be produced and all the combinations that will give this maximum. **[4]**

11 A small factory makes two types of components, X and Y. Each component of type X requires materials costing £18 and each component of type Y requires materials costing £11. In each week materials worth £200 are available.

Each component of type X takes 7 man hours to finish and each component of type Y takes 6 man hours to finish. There are 84 man hours available each week.

Components cannot be left part-finished at the end of the week. In addition, in order to satisfy customer demands, at least 2 of each type are to be made each week.

(i) The factory produces x components of type X and y components of type Y each week. Write down four inequalities for x and y. [4]

(ii) On a graph draw suitable lines and shade the region that the inequalities do not allow. (Take 1 cm = 1 component on each axis.) [5]

(iii) If all components made are sold and the profit on each component of type X is £70 and on each component of type Y is £50, find from your graph the optimal number of each that should be made and the total profit per week. [3]

8 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$3x + 2y \leqslant 18$$
$$y \leqslant 3x$$
$$y \geqslant 0$$

[5]

(ii) Find the maximum value of $x + 2y$ subject to these conditions. [2]

10 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$2x + 3y \leqslant 12$$
$$2x + y \leqslant 8$$
$$y \geqslant 0$$
$$x \geqslant 0$$

[5]

(ii) Find the maximum value of $x + 3y$ subject to these conditions. [2]

12 A furniture manufacturer produces tables and chairs.

In each week the following constraints apply.

- There are 24 workers, each working for 40 hours (i.e. there are 960 worker-hours available).
- There is a maximum of £1800 available for the purchase of materials.
- Each table requires £30 worth of materials and 12 worker-hours.
- Each chair requires £10 worth of materials and 6 worker-hours.
- It is necessary to make at least 3 times as many chairs as tables.

Let x be the number of tables produced each week and y be the number of chairs produced each week.

(i) Show that the worker-hour constraint reduces to the inequality $2x + y \leqslant 160$. [2]

(ii) Find the inequality relating to the cost of materials constraint and the inequality relating to the numbers of tables and chairs. [3]

(iii) Plot these three inequalities on a graph, using 1 cm to represent 10 tables on the x-axis and 1 cm to represent 10 chairs on the y-axis. Indicate the region for which these inequalities hold. You should shade the region which is **not** required. [4]

When finished, each table is sold for a profit of £20 and each chair is sold for a profit of £5.

(iv) The manufacturer wishes to maximise the profit. Explain why the objective function is given by $P = 20x + 5y$. [1]

(v) Find the number of tables and chairs that should be made in order to maximise the profit. [2]